

Gilbert Damping in Magnetic Multilayers

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We study the enhancement of the ferromagnetic relaxation rate in thin films due to the adjacent normal metal layers. Using linear response theory, we derive the dissipative torque produced by the $s-d$ exchange interaction at the ferromagnet-normal metal interface. For a slow precession, the enhancement of Gilbert damping constant is proportional to the square of the $s-d$ exchange constant times the zero-frequency limit of the frequency derivative of the local dynamic spin susceptibility of the normal metal at the interface. Electron-electron interactions increase the relaxation rate by the Stoner factor squared. We attribute the large anisotropic enhancements of the relaxation rate observed recently in multilayers containing palladium to this mechanism. For free electrons, the present theory compares favorably with recent spin-pumping result of Tserkovnyak et al. [Phys. Rev. Lett. **88**,117601 (2002)].

I. INTRODUCTION

Ferromagnetic multilayers have attracted much attention recently because of their applications in spintronics and high density magnetic recording devices. The present paper is concerned with magnetic relaxation in a ferromagnetic film (F) imbedded between nonmagnetic metallic layers (N). In particular, we study the enhancement of the Gilbert damping in $N/F/N$ sandwiches as compared with that of a single ferromagnetic film. The Gilbert damping constant G is defined by the Landau-Lifshitz-Gilbert (LLG) equation of motion [1,2]

$$\frac{1}{|\gamma|} \frac{\partial \mathbf{M}}{\partial t} = -[\mathbf{M} \times \mathbf{H}_{eff}] + \frac{G}{\gamma^2 M_s^2} \left(\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} \right), \quad (1)$$

where \mathbf{M} is the magnetization vector, M_s is the saturation magnetization, and \mathbf{H}_{eff} is the effective field which is given by $\mathbf{H}_{eff} = -\partial E / \partial \mathbf{M}$ where E is the Gibbs free energy. The gyromagnetic ratio γ is a negative quantity $-g\mu_B/\hbar$ where g is the spectroscopic splitting factor, and μ_B is the Bohr magneton. The second term on the R. H. S. of Eq.(1) represents the dissipative torque, the magnitude of which is proportional to G . The LLG equation describes well both the static and the dynamic properties of ultrathin ferromagnetic films and the $N/F/N$ sandwiches (for a review see Ref.[3]). The experimental and theoretical aspects of spin relaxation in multilayers are covered in Ref.[4].

Recent studies of the FMR linewidth for ultrathin films [5,6] show that the constant G is enhanced when a nonmagnetic metal is deposited on the ferromagnetic film. This effect has been predicted in 1996 by Berger [7] in his study of spin transfer between the polarized conduction electrons and the ferromagnetic film. Berger shows that

there is an enhanced electron-magnon scattering caused by isotropic $s-d$ exchange taking place at the F/N interface. Being a surface effect, the enhancement of the Gilbert constant is inversely proportional to the thickness of the ferromagnetic film. This unique feature is observed in recent measurements of the additional FMR linewidth on permalloy-normal metal sandwiches [8], and on the double ferromagnetic layer structures as well [9]. It should be mentioned that already in 1987 a study of FMR linewidth showed an appreciable increase in the Gilbert damping with decreasing thickness of the Fe ultrathin films grown on bulk $Ag(001)$ substrates [10].

A novel mechanism for additional Gilbert damping in $N/F/N$ structures has been recently proposed by Tserkovnyak et al.[11]. These authors calculate the spin current pumped through the $N-F$ contact by the precession of the magnetization vector $\mathbf{M}(t)$. The theory is based on extending the scattering approach of parametric charge pumping by Brouwer [12] to spin pumping.

Like the theory of Berger [7], the additional damping of Ref.[11] scales inversely with the thickness of the ferromagnetic film indicating that only the F/N interface is involved. However, the expression for excess G , which we call G' , differs considerably from that of Ref.[7]. In particular, G' vanishes with vanishing $s-d$ exchange splitting. An attractive feature of this theory is that it links G' to the transport properties of the interface. Due to exchange polarization of the F/N contact, the reflection (r) and transmission (t) coefficients at the interface depend on the orientation of the conduction electron spin with respect to the magnetization direction of the ferromagnet. The formula for G' involves differences such as $\Delta r = r^\uparrow - r^\downarrow$. Interestingly, similar quantities play a role in the theory of interlayer magnetic coupling by Bruno [13]. For instance, the Ruderman-Kittel-Kasuya-Yosida (RKKY) coupling [14] between two-dimensional layers can be obtained by calculating the interlayer coupling energy in terms of the reflection coefficients r^\uparrow and r^\downarrow of the layers. These coefficients are obtained by solving

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a simple problem of scattering by δ -function potential. In the limit of weak exchange splitting (compared to the Fermi energy), the derived interlayer coupling agrees with the RKKY result of Yafet [15].

These considerations prompt us to take another look at the theory of enhanced relaxation in multilayer. The fact that the RKKY theory [13] involves transport properties at the interface similar to the spin pump theory of G' [11] suggests that a suitable generalization of RKKY theory to time-dependent magnetization $\mathbf{M}(t)$ may unravel the needed dissipative torque of Eq.(1). We note that the standard approach to RKKY coupling is to use a linear response theory [14,16], and calculate the conduction electron spin density induced by the contact exchange potential. In applications to interlayer coupling between ferromagnetic layers with time-independent magnetization vectors, it is the static spin susceptibility of the electron gas which determined the coupling. In the present paper, we consider a response to a slowly varying time-dependent $s-d$ exchange potential.

Owing to the dissipative part of the spin susceptibility, the spin density induced by the precession of $\mathbf{M}(t)$ will have a component that is out of phase with $\mathbf{M}(t)$. Hence, such a component will have a time-dependence given by $d\mathbf{M}/dt$. The induced spin density has both local and nonlocal consequences for the dynamics of multilayers.

First, let us focus on the local effect in the $N/F/N$ system. It is instructive to invoke the analogy to radiation damping of charged particle in classical electrodynamics [17]. Thus, we put the dissipative torque of Eq.(1) in correspondence with the radiation reaction force acting on the particle. The first term of Eq.(1) corresponds to the external force. In view of this analogy, we rewrite the dissipative torque in terms of a reaction field \mathbf{H}_r .

$$\frac{G'}{\gamma^2 M_s^2} [\mathbf{M} \times \frac{d\mathbf{M}}{dt}] = -[\mathbf{M} \times \mathbf{H}_r^{(d)}] \quad (2)$$

where $\mathbf{H}_r^{(d)}$ represents the dissipative part of \mathbf{H}_r . For the $N/F/N$ system shown in Fig.1., we find a reaction field (see Appendix A)

$$\mathbf{H}_r(t) = \frac{2Ja}{\gamma d} \langle \mathbf{s}(x=0, t) \rangle \quad (3)$$

where J is the $s-d$ exchange coupling constant, a is of order of the lattice constant, and d is the width of the ferromagnetic film. The quantity $\langle \mathbf{s}(x=0, t) \rangle$ is the spin density induced at the F/N interface by the $s-d$ exchange interaction (x being the distance from the interface). The expression (3) is quite general in the sense that it can be used for both ballistic and diffusive cases. In the present paper we confine ourselves to the ballistic case and calculate the induced spin density using linear response theory [16].

Assuming slow precession of $\mathbf{M}(t)$, we find two contributions: One that is proportional to $\mathbf{M}(t)$ with a coefficient given by the real part of the local spin susceptibility

at zero frequency. If the spin susceptibility is anisotropic, this term leads to an anisotropic shift of the FMR frequency. For isotropic susceptibility, the quantity on the R.H.S. of Eq.(3) is a vector parallel to $\mathbf{M}(t)$ and the corresponding torque (2) vanishes.

The second contribution to $\langle \mathbf{s}(x=0, t) \rangle$ is proportional to $d\mathbf{M}/dt$. The coefficient of proportionality is the frequency derivative of the imaginary part of the susceptibility taken at $\omega = 0$. Like the real part, this quantity is generally anisotropic. According to Eq.(2), it produces a dissipative torque leading to an anisotropic G' . Explicit evaluation of this term for an isotropic noninteracting electron gas yields a formula for G' which compares favorably with the spin pump theory of Tserkovnyak et al.[11].

The proposed formulation allows us to incorporate interactions between the electrons in the N region. The generalized Hartree-Fock approximation shows that the \mathbf{q}, ω dependent spin susceptibility of the interacting electron gas is enhanced compared to the free electron case [16]. Using these results, we find that the part of \mathbf{H}_r that contributes to the FMR frequency shift is enhanced by the Stoner factor

$$S_E = [1 - UN(\epsilon_F)]^{-1} \quad (4)$$

where U is the screened intraatomic Coulomb interaction and $N(\epsilon_F)$ is the electron density of states, per atom, at the Fermi level [16]. On the other hand, we find that G' is enhanced by a factor of S_E^2 . The Stoner enhancement is thought to be large in metals such as palladium and its alloys. Recent results using multilayers with Pd layers as a spacer show a significant enhancement of interface damping exhibiting a fourfold anisotropy in keeping with the present theory [18].

Let us turn to the nonlocal consequences of the dynamically induced spin density. This is relevant in the case of two time-dependent magnetizations $\mathbf{M}_1(t)$ and $\mathbf{M}_2(t)$ separated by a nonmagnetic spacer. Similar to the local effect, the magnetization $\mathbf{M}_1(t)$ at x_1 induces at the position x_2 of $\mathbf{M}_2(t)$ a spin density oscillation with both the in phase ($\propto \mathbf{M}_1(t)$) and the out of phase component ($\propto d\mathbf{M}_1/dt$). The interaction of the in-phase component with $\mathbf{M}_2(t)$ yields the standard interlayer RKKY coupling [15]. The out of phase component leads to a dissipative interaction of the form $d\mathbf{M}_1(t)/dt \cdot \mathbf{M}_2(t) f(x_2 - x_1)$. A similar form, with the indices 1 and 2 interchanged, follows by taking into account the spin density induced at x_1 by precession of $\mathbf{M}_2(t)$. Such terms may be of importance in FMR studies involving dynamically coupled magnetic films. However, these topics are beyond the scope of the present investigation which focuses on dynamics of single ferromagnetic film in contact with nonmagnetic metal.

The paper is organized as follows. In Sec.II we derive the spin density induced by a two-dimensional layer of precessing spins, and the corresponding reaction field. Sec.III focuses on the dissipative part of the reaction field, and the excess damping G' for an isotropic electron gas.

An expression for G' in a free electron model is derived for $N/F/N$ structure with N layers of both infinite and finite thickness. The enhancement of G' due to electron-electron interactions is considered within the generalized Hartree-Fock approximation. In Sec.IV we establish a relation between the spin pumping theory of Ref.[11] and our free electron result for G' . The role of spin sink in theories of Gilbert damping is discussed in Sec.V. The reaction field for the $N/F/N$ structure is derived in Appendix A. The role of spin relaxation in the theory of G' is considered in Appendix B.

II. DYNAMIC RKKY

We consider a two-dimensional layer of aligned spins imbedded in normal metal. Notice that a similar model was used by Yafet [15] to calculate the interlayer coupling for time-independent magnetizations. Here we take into account the time-dependence of the precessing magnetization.

Our task is to derive the conduction electron spin density induced by the $s-d$ exchange interaction taking place at the magnetic layer. The Hamiltonian of the conduction electrons is

$$\hat{H} = \hat{H}_0 - J \sum_i^{sheet} \int d^3r \mathbf{S}^{(i)}(t) \cdot \hat{\mathbf{s}}(\mathbf{r}) \delta^3(\mathbf{r} - \mathbf{r}_i) \quad (5)$$

where \hat{H}_0 represents the Hamiltonian of the electrons in the absence of the ferromagnetic layer, and the second term is the $s-d$ exchange interaction with the layer.

J is the $s-d$ exchange coupling constant, $\mathbf{S}^{(i)}(t)$ is the classical spin at the location \mathbf{r}_i , and $\hat{\mathbf{s}}(\mathbf{r})$ is the spin density operator for the electrons in normal metal.

For a uniform precession of aligned spins, we have

$$\mathbf{S}^{(i)}(t) = \frac{\Omega}{\gamma} \mathbf{M}(t) \quad (6)$$

where Ω is the volume of the unit cell.

The second term of Eq. (5) acts as a time-dependent perturbation that will be treated using the linear response theory [16]. Thus, the expectation value of the μ -component of the induced spin density is

$$\langle s_\mu(\mathbf{r}, t) \rangle = J \sum_i^{sheet} \int d^3r' \int_{-\infty}^{\infty} dt' \chi_{\mu\nu}(\mathbf{r}t, \mathbf{r}'t') \times S_\nu^{(i)}(t') \delta^3(\mathbf{r}' - \mathbf{r}_i) \quad (7)$$

where

$$\chi_{\mu\nu}(\mathbf{r}t, \mathbf{r}'t') = \frac{i}{\hbar} \Theta(t - t') \langle [\hat{s}_\mu(\mathbf{r}, t), \hat{s}_\nu(\mathbf{r}', t')] \rangle \quad (8)$$

is the retarded spin correlation function (susceptibility) [16], $\Theta(t - t')$ being the unit step function.

Now, we evaluate the R.H.S. of Eq.(7) assuming a slow precession. This approximation is valid as long as the precessional frequency is small compared to the relevant excitation frequencies of the conduction electrons. We note that the spin pump theory requires a similar condition to hold [11, 12].

Performing the \mathbf{r}' -integration, and applying the commutation law for the convolution, Eq.(7) reads

$$\langle s_\mu(\mathbf{r}, t) \rangle = J \sum_i^{sheet} \int_{-\infty}^{\infty} dt' S_\nu^{(i)}(t - t') \chi_{\mu\nu}(\mathbf{r}, \mathbf{r}_i, t') \quad (9)$$

For slow precession, we write

$$S_\nu^{(i)}(t - t') \simeq S_\nu^{(i)}(t) - t' \frac{dS_\nu^{(i)}(t)}{dt} \quad (10)$$

Introducing this expansion into Eq.(9), we obtain with use of Eq.(6)

$$\langle s_\mu(\mathbf{r}, t) \rangle \simeq \frac{J\Omega}{\gamma} \lim_{\omega \rightarrow 0} \left[M_\nu(t) \sum_i^{sheet} \chi_{\mu\nu}(\mathbf{r}, \mathbf{r}_i; \omega) - \frac{\partial M_\nu(t)}{\partial t} \sum_i^{sheet} \frac{\partial \text{Im} \chi_{\mu\nu}(\mathbf{r}, \mathbf{r}_i, \omega)}{\partial \omega} \right] \quad (11)$$

where

$$\chi_{\mu\nu}(\mathbf{r}, \mathbf{r}_i, \omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} \chi_{\mu\nu}(\mathbf{r}, \mathbf{r}_i, t) \quad (12)$$

We note that the second term on the R.H.S. of Eq.(11) follows by letting the derivative of the real part of the susceptibility with respect to ω equal to zero at $\omega = 0$. This is consistent with the reality condition implying that that the real part of the susceptibility is an even function of ω . The first term on the R.H.S. of Eq.(11) corresponds to the static RKKY result of Yafet [15]. The second term is the dynamic generalization of RKKY for a slowly precessing magnetization.

For an infinite medium, the sheet sums in eq. (11) can be evaluated by invoking translational invariance and Fourier transforming $\chi_{\mu\nu}(\mathbf{r}, \mathbf{r}_i, \omega)$ to \mathbf{q} -space [13]. Thus, we define a generic sum

$$X_{\mu\nu}(\mathbf{r}, \omega) = \sum_i^{sheet} \chi_{\mu\nu}(\mathbf{r}, \mathbf{r}_i, \omega) = \int \frac{d^3q}{(2\pi)^3} \times \sum_i^{sheet} \exp[i\mathbf{q} \cdot (\mathbf{r} - \mathbf{r}_i)] \chi_{\mu\nu}(\mathbf{q}, \omega) \quad (13)$$

from which both terms of Eq.(11) can be deduced.

To perform the q -integral on the R.H.S. of this equation, we set

$$\mathbf{q} = \mathbf{q}_{\parallel} + \mathbf{q}_{\perp} \quad (14)$$

where \mathbf{q}_{\parallel} is confined to the sheet, and \mathbf{q}_{\perp} is perpendicular to the sheet. For a square sheet of area L^2 , we have

$$\int \frac{d^3 q}{(2\pi)^3} = \int \frac{d^2 q_{\parallel}}{(2\pi)^2} \int \frac{dq_{\perp}}{2\pi} = \frac{1}{L^2} \sum_{q_{\parallel}} \int \frac{dq_{\perp}}{2\pi} \quad (15)$$

Assuming a continuous distribution of spins, the sheet sum in Eq. (13) is given by

$$\sum_i^{sheet} \exp[-i(\mathbf{q}_{\parallel} + \mathbf{q}_{\perp}) \cdot \mathbf{r}_i] = N_s \delta_{\mathbf{q}_{\parallel}, 0} \quad (16)$$

where N_s is number of spins in the sheet. Using Eqs. (14-16), the R.H.S. of Eq.(13) is evaluated with the result

$$\begin{aligned} X_{\mu\nu}(\mathbf{r}, \omega) &= \frac{N_s}{L^2} \int \frac{dq_{\perp}}{2\pi} \exp(i\mathbf{q}_{\perp} \cdot \mathbf{r}) \chi_{\mu\nu}(\mathbf{q}_{\perp}, \omega) \\ &= n_s \int \frac{dq_{\perp}}{2\pi} \exp(iq_{\perp} x) \chi_{\mu\nu}(q_{\perp}, \omega) \end{aligned} \quad (17)$$

where $n_s = N_s/L^2$ is the sheet density. Owing to the symmetry of the model, the quantity $X_{\mu\nu}(\mathbf{r}, \omega)$ depends only on the distance x from the sheet.

Also, the induced spin density is a function of x . Using Eqs.(11), (13) and (17), we obtain

$$\begin{aligned} \langle s_{\mu}(x, t) \rangle &= \frac{J\Omega}{\gamma} \lim_{\omega \rightarrow 0} \left[X_{\mu\nu}(x, \omega) M_{\nu}(t) \right. \\ &\quad \left. - \frac{\partial \text{Im} X_{\mu\nu}(x, \omega)}{\partial \omega} \frac{dM_{\nu}(t)}{dt} \right] \end{aligned} \quad (18)$$

Using this result and Eq.(17) in Eq.(3), the μ -component of the reaction field is

$$\begin{aligned} H_{r,\mu}(t) &\simeq \frac{2J^2\Omega n_s}{\gamma^2 d} \lim_{\omega \rightarrow 0} \left[\int_{-\infty}^{\infty} \frac{dq}{2\pi} \chi_{\mu\nu}(q, \omega) M_{\nu}(t) \right. \\ &\quad \left. - \frac{\partial}{\partial \omega} \int_{-\infty}^{\infty} \frac{dq}{2\pi} \text{Im} \chi_{\mu\nu}(q, \omega) \frac{dM_{\nu}(t)}{dt} \right] \end{aligned} \quad (19)$$

The first term on the R.H.S of this equation contributes to the torque $[\mathbf{M} \times \mathbf{H}_{eff}]$ only if $\chi_{\mu\nu}$ is anisotropic. In this case an anisotropic shift of the FMR frequency ensues. On the other hand, the second term contributes to the dissipative torque that is non vanishing for both isotropic and anisotropic susceptibility.

III. GILBERT DAMPING

In what follows, we consider the dissipative torque for an isotropic electron gas. Thus, $\chi_{\mu\nu}(\mathbf{q}, \omega) = \chi(\mathbf{q}, \omega) \delta_{\mu\nu}$,

and the dissipative part of the reaction field $\mathbf{H}_r^{(d)}$ is according to Eq.(19) given by

$$\begin{aligned} \mathbf{H}_r^{(d)}(t) &\simeq -\frac{2J^2\Omega n_s}{\gamma^2 d} \\ &\times \lim_{\omega \rightarrow 0} \left[\frac{\partial}{\partial \omega} \int_{-\infty}^{\infty} \frac{dq}{2\pi} \text{Im} \chi(q, \omega) \right] \frac{d\mathbf{M}(t)}{dt} \end{aligned} \quad (20)$$

Introducing this result in the R.H.S. of Eq.(2), we obtain the damping enhancement constant G'

$$G' \simeq 2J^2\Omega n_s M_s^2 \left(\frac{a}{d}\right) \lim_{\omega \rightarrow 0} \left[\frac{\partial}{\partial \omega} \int_{-\infty}^{\infty} \frac{dq}{2\pi} \text{Im} \chi(q, \omega) \right] \quad (21)$$

A. Independent electrons

First, we evaluate the expression (21) by disregarding the electron-electron interaction in the N regions. However, a finite splitting Δ of the \uparrow and \downarrow spin bands is assumed. The external magnetic field of the FMR experiment is one source of this splitting. For a system of infinite size, this splitting establishes a lower cutoff on the wave vector q . As shown below, this cutoff is essential to prevent the logarithmic divergence of Eq.(21). Due to the spin splitting, the susceptibility develops some anisotropy. Since transverse components of the reaction field (19) contribute to the dissipative torque, we need to use in Eq.(21) the transverse susceptibility [16]

$$\chi_T^{(0)}(\mathbf{q}, \omega) = \frac{\hbar^2}{4} \int \frac{d^3 k}{(2\pi)^3} \frac{f_{k+q\downarrow} - f_{k\uparrow}}{\hbar\omega - \epsilon_{k+q\downarrow} + \epsilon_{k\uparrow} + i\eta} \quad (22)$$

where

$$\epsilon_{k+q\downarrow} - \epsilon_{k\uparrow} \simeq \frac{\hbar^2 \mathbf{k} \cdot \mathbf{q}}{m} + \Delta \quad (23)$$

Expanding the Fermi functions, we have

$$f_{k+q\downarrow} - f_{k\uparrow} \simeq \frac{\partial f}{\partial \epsilon_k} \left(\frac{\hbar^2 \mathbf{k} \cdot \mathbf{q}}{m} + \Delta \right) \quad (24)$$

Using Eqs.(23) and (24), the imaginary part of Eq.(22) becomes

$$\begin{aligned} \text{Im} \chi_T^{(0)}(\mathbf{q}, \omega) &\simeq \frac{-\hbar^2}{16\pi} \int_0^{\infty} dk k^2 \frac{\partial f}{\partial \epsilon_k} \int_0^{\pi} d\theta \sin \theta \\ &\times \left(\Delta + \frac{\hbar^2 \mathbf{k} \cdot \mathbf{q}}{m} \right) \delta \left(\hbar\omega - \Delta - \frac{\hbar^2 \mathbf{k} \cdot \mathbf{q}}{m} \right) \end{aligned} \quad (25)$$

where we used polar coordinates to perform the \mathbf{k} -integration. Performing first the integration over polar angle θ , we have

$$\begin{aligned} \int_0^{\pi} d\theta \sin \theta \left(\Delta + \frac{\hbar^2 \mathbf{k} \cdot \mathbf{q}}{m} \right) \delta \left(\hbar\omega - \Delta - \frac{\hbar^2 \mathbf{k} \cdot \mathbf{q}}{m} \right) \\ = \frac{m\omega}{\hbar k q} \Theta \left(k - \frac{|\hbar\omega - \Delta| m}{\hbar^2 q} \right) \end{aligned} \quad (26)$$

Introducing this result into Eq.(25), and converting the k -integral to ϵ_k -integration, we obtain at $T = 0$

$$\begin{aligned} \text{Im}\chi_T^{(0)}(q, \omega) &= \frac{m^2\omega}{16\pi\hbar q} \int_0^\infty d\epsilon \delta(\epsilon - \epsilon_F) \\ &\times \Theta\left(\sqrt{\frac{2m\epsilon}{\hbar^2}} - \frac{|\hbar\omega - \Delta|m}{\hbar^2 q}\right) \\ &= \frac{m^2\omega}{16\pi\hbar q} \Theta\left(k_F - \frac{|\hbar\omega - \Delta|m}{\hbar^2 q}\right) \end{aligned} \quad (27)$$

The unit step function on the R.H.S. of this equation equals one for $q > q_1$, and zero for $q < q_1$ where

$$q_1 = \frac{|\hbar\omega - \Delta|m}{\hbar^2 k_F} \quad (28)$$

Thus, q_1 acts as a lower cutoff in the q -integral of $\text{Im}\chi(q, \omega)$. Since $|\hbar\omega - \Delta| \ll \epsilon_F$, the upper cutoff is given by $q_2 \approx 2k_F$. We then get using Eq.(27)

$$\begin{aligned} \int_{-\infty}^\infty \frac{dq}{2\pi} \text{Im}\chi_T^{(0)}(q, \omega) &\approx \frac{m^2\omega}{16\pi^2\hbar} \\ &\times \int_{q_1}^{q_2} \frac{dq}{q} = \frac{m^2\omega}{16\pi^2\hbar} \ln \frac{4\epsilon_F}{|\hbar\omega - \Delta|} \end{aligned} \quad (29)$$

Introducing this result into Eq.(21), we obtain

$$G' \simeq \frac{(JM_s a m)^2}{8\pi^2 \hbar d} \ln \frac{4\epsilon_F}{\Delta} \quad (30)$$

where we assumed a cubic lattice to write $\Omega n_s a = a^2$. Strictly speaking, G' in this equation is not a simple Gilbert damping since its strength depends on the applied magnetic field, but it is a weak dependence.

When Δ vanishes, G' exhibits a logarithmic divergence. This is due to the fact that precessing magnetization radiates into the N -region a very large number of electron-hole pairs of low energy. This is reminiscent of the infrared divergence in quantum electrodynamics where a large number of low energy photons is radiated. Endowing the photon with a small mass cures this divergence [19]. In an analogous way, a nonzero spin splitting of the electron energy levels suppresses the number of low energy electron-hole pairs that are generated by the precessing magnetization.

For $\Delta \simeq g\mu_B H$, we have

$$\ln \frac{4\epsilon_F}{\Delta} = \ln \left(\frac{2k_F^2 \hbar c}{eH} \right) \approx \ln \frac{l_H}{a} \quad (31)$$

We see that it is the magnetic length

$$l_H = \left(\frac{\hbar c}{eH} \right)^{\frac{1}{2}} \quad (32)$$

which plays the role of q_1^{-1} in case of infinitely wide N layers. Taking $H = 10^4$ gauss and putting $k_F \simeq 1/a$, we

obtain from Eq.(32) $l_H \simeq 4 \times 10^2 a$. In real samples, the N layers are usually considerably thinner than this length. As shown in the next subsection, the relevant cutoff is then determined by the boundary conditions (b.c.) at the outer surfaces of the sandwich.

It is interesting that spin relaxation of the conduction electrons can provide an effective gap and a finite cut-off length even in the limit of infinitely thick N layers. As shown in Appendix B, the resulting cutoff length is $q_1^{-1} \approx v_F \tau_s$ where τ_s is the spin-relaxation time. The corresponding value of the effective gap is $\Delta_{eff} \approx \hbar/\tau_s$. The same quantity determines the linewidth of the electron paramagnetic resonance of conduction electrons [23]. Its magnitude is presumably not negligible in comparison with the magnetic splitting $g\mu_B H$.

B. Finite normal layers

We consider a ferromagnetic sheet imbedded between two N layers each of thickness D . Strictly speaking, in a finite system, the translational invariance invoked in the evaluation of the sheet sum (13) is broken. This complicates the calculation of the induced spin density. Nevertheless, a simplified approximate evaluation of $X_{\mu\nu}(x, \omega)$ can be carried out if $x \simeq 0$. In this case, the translational invariance is restored locally since the boundary at $x = D$ plays a small role. The b.c. at $x = D$ is taken in the form

$$\langle s_\mu(x, t) \rangle|_{x=D} = 0 \quad (33)$$

This condition follows from assuming that there is an infinite potential step at the normal metal-vacuum boundary. Thus, the N -electron wave functions are forced to zero at the boundary and so is the magnetization density. Applying this b.c. to Eq.(18), the finite- D version of Eq.(17) reads

$$X_{\mu\nu}(x, \omega) \simeq \frac{n_s}{D} \sum_{n=1}^{n_{max}} \cos(q_n x) \chi_{\mu\nu}(q_n, \omega) \quad (34)$$

where $q_n = \pi(n - 1/2)/D$, and $n_{max} \simeq D/a$ since it is the lattice spacing which determines the highest value of q_n . In view of this, Eq.(21) is changed to

$$G' \simeq \frac{2a}{dD} J^2 \Omega n_s M_s^2 \lim_{\omega \rightarrow 0} \left[\frac{\partial}{\partial \omega} \sum_{n=1}^{n_{max}} \text{Im}\chi(q_n, \omega) \right] \quad (35)$$

For noninteracting electrons without spin splitting, we have

$$\sum_{n=1}^{n_{max}} \text{Im}\chi(q_n, \omega) \simeq \frac{m^2\omega}{16\pi\hbar} \sum_{n=1}^{n_{max}} q_n^{-1} \quad (36)$$

For $D/a \gg 1$, we use the definition of the Euler constant $\gamma_E \simeq 1.78$ to obtain

$$\sum_{n=1}^{n_{max}} q_n^{-1} \approx \frac{D}{\pi} \ln(4\gamma_E n_{max}) \quad (37)$$

Using Eqs.(35-37), we obtain

$$G' \approx \frac{(JM_s a m)^2}{8\pi^2 \hbar d} \ln(D/a) \quad (38)$$

As expected, the boundary conditions in a finite slab imply a cutoff $q_1 \approx D^{-1}$.

We now make an order of magnitude estimate of Eq.(38) for an iron film of thickness $d = D = 10a$ where $a = 4 \times 10^{-8} \text{cm}$. The constant J can be estimated by relating it to the atomic exchange integral J_{sd}

$$J \approx \frac{2J_{sd}\Omega}{\hbar^2} \quad (39)$$

Taking $J_{sd} = 0.1 \text{eV}$, and $M_s = 1.7 \times 10^3 \text{gauss}$, Eq.(38) yields $G' \approx 10^8 \text{s}^{-1}$. This agrees with the interface damping observed recently in the double layer structure by Urban et al.[9]. It should be pointed out that the second ferromagnetic layer in this experiment plays a crucial role in establishing the spin sink needed to prevent spin accumulation in the N layers(see Sec.V.)

C. Electron-electron interactions

We now calculate G' by taking into account interactions between electrons in the normal metal. The generalized Hartree-Fock approximation for the Hubbard model yields the following expression for the transverse susceptibility [20]

$$\chi_T(q, \omega) = \frac{\chi_T^{(0)}(q, \omega)}{1 - \tilde{U}\chi_T^{(0)}(q, \omega)} \quad (40)$$

where $\tilde{U} = 4\Omega U/\hbar^2$, and U is the screened intraatomic Coulomb energy. Using this formula, we have

$$\lim_{\omega \rightarrow 0} \frac{\partial}{\partial \omega} \left[\text{Im} \chi_T(q, \omega) \right] = \left[1 - \tilde{U}\chi_T^{(0)}(q, 0) \right]^{-2} \times \lim_{\omega \rightarrow 0} \frac{\partial}{\partial \omega} \left[\text{Im} \chi_T^{(0)}(q, \omega) \right] \quad (41)$$

To simplify the evaluation of the q -integral of this quantity, we take advantage of the weak dependence of the static susceptibility on q for $q < q_F$, and make the approximation

$$\chi_T^{(0)}(q, 0) \approx \chi_T^{(0)}(0, 0) = \frac{\hbar^2}{4\Omega} N(\epsilon_F) \quad (42)$$

where $N(\epsilon_F)$ is the density of states, per atom, at the Fermi energy. Using Eqs.(41) and (42) in Eq.(35), we

obtain the enhancement, G' , for interacting electrons in finite N layers

$$G' \approx \frac{(JM_s a m S_E)^2}{8\pi^2 \hbar d} \ln(D/a) \quad (43)$$

where S_E is the Stoner factor defined in Eq.(4). For palladium, we have $S_E \approx 10$. Thus, large values of G' are expected for sandwiches containing Pd as normal layers. Mizukami et al.[8] measured the Gilbert damping constant in $N/F/N$ sandwiches with F being a thin film of permalloy (Py). These measurements show that G' for $Pd/Py/Pd$ system is well above that for the $Cu/Py/Cu$ system. However, it is about twice as smaller than G' for $Pt/Py/Pt$. This seems to contradict our Eq.(43). We attribute this disagreement to spin accumulation in the normal layers. Since the spin-orbit coupling constant of Pt is larger (by a factor of 3) than that for Pd , the spin lattice relaxation rate in Pt is an order of magnitude stronger than that in Pd . As pointed out by Tserkovnyak et al [11], the spin accumulation takes place when the spin relaxation rate is small. The fact that experimental ratio G'_{Pt}/G'_{Pd} is less than 10 indicates that Stoner enhancement in Pd is not excluded.

More convincing evidence of Stoner enhancement in interface damping comes from recent FMR studies on $20Au/40Fe/40Au/3Pd/[Fe/Pd]_5/14Fe/GaAs(001)$ samples [18]. Compared to a single layer structure, these samples show a G' that is enhanced by a factor of four and exhibits a strong fourfold in-plane anisotropy. Apparently, the presence of a second Fe layer provides an efficient spin sink. Thus G' is determined by the exchange enhanced susceptibility rather than the bottleneck due to a weak spin-lattice relaxation in the N -layers.

We now digress, for a moment, to consider the enhancement of the FMR frequency shift due to interactions. Applying Eq.(40) to the static anisotropic susceptibility, and making the approximation (42), we have $\chi_{\mu\nu}(q, 0) \approx S_E \chi_{\mu\nu}^{(0)}(q, 0)$. Using this result in Eq.(19), we see that the frequency shift for the interacting electrons is S_E times that for the independent electrons. This prediction could be used, in conjunction with the data for the anisotropic G' to clarify experimentally the role of interactions in the FMR of multilayers.

IV. RELATION TO SPIN-PUMPING THEORY

We now show that for free electrons there is a similarity between our formula (30) for G' and the spin-pumping theory of Tserkovnyak et al. [11]. According to these authors, the excess damping produced by pumping of spins into adjacent N -layers is $G' = \gamma M_s \alpha'$ where

$$\alpha' = \frac{g_L \mu_B (A_r^{(L)} + A_r^{(R)})}{4\pi M_s L^2 d} \quad (44)$$

where g_L is the Landé factor, μ_B is the Bohr magneton, and $A_r^{(L)}$, $A_r^{(R)}$ are the interface parameters for the left,

right N -Layers, respectively. In terms of the elements of the 2×2 scattering matrix, for a symmetric $N/F/N$ sandwich, these parameters are

$$A_r^{(L)} = A_r^{(R)} = A_r \quad (45)$$

$$= \frac{1}{2} \sum_{mn} \{|r_{mn}^\uparrow - r_{mn}^\downarrow|^2 + |t_{mn}'^\uparrow - t_{mn}'^\downarrow|^2\}$$

where $(r_{mn}^\uparrow, r_{mn}^\downarrow)$ and $(t_{mn}'^\uparrow, t_{mn}'^\downarrow)$ are the reflection and transmission coefficients for electrons with up and down spins. The expression (45) is to be evaluated with the transverse modes (m, n) taken at the Fermi energy.

Following Bruno [13], we consider the scattering of N -electrons by a ferromagnetic monolayer. Due to the conservation of transverse momentum, $r_{mn} = r_m \delta_{mn}$ where $r_m = r_0(k_\perp)$, k_\perp being the component of the electron wave vector perpendicular to the monolayer.

The reflection coefficients $r_0(k_\perp)$ are found by solving the one-dimensional scattering problem for the potential

$$v(x) = v_0 \delta(x) \quad (46)$$

where v_0 is given by the interface coupling constant J , and by the magnitude of the atomic spin S of the ferromagnet

$$v_0 = \pm \frac{\hbar}{2} J S n_s \quad (47)$$

where the $(+, -)$ signs correspond to the (\downarrow, \uparrow) electron spins, respectively.

The reflection coefficients for this problem are [13]

$$r_0^\uparrow = \frac{-i\beta}{k_\perp + i\beta} \quad (48)$$

$$r_0^\downarrow = \frac{i\beta}{k_\perp - i\beta}$$

where

$$\beta = \frac{mv_0}{\hbar^2} \quad (49)$$

The transmission coefficients are [20]

$$t_0^\uparrow = \frac{ik_\perp}{ik_\perp - \beta} \quad (50)$$

$$t_0^\downarrow = \frac{ik_\perp}{ik_\perp + \beta}$$

Eqs.(48) and (50) imply

$$|r_0^\uparrow - r_0^\downarrow|_{\epsilon_F}^2 = |t_0^\uparrow - t_0^\downarrow|_{\epsilon_F}^2 \quad (51)$$

$$= \frac{4\beta^2(k_F^2 - k_\parallel^2)}{(k_F^2 - k_\parallel^2 + \beta^2)^2}$$

where we applied the identity $(k_\parallel^2 + k_\perp^2)_{\epsilon_F} = k_F^2$. Using this result, the transverse mode sum (45) becomes a sum

over the in-plane wave vectors \mathbf{k}_\parallel . Converting the \mathbf{k}_\parallel -sum to a two-dimensional integral, we have

$$A_r \simeq \frac{L^2}{2\pi} \int_0^{k_F} dk_\parallel k_\parallel \frac{4\beta^2(k_F^2 - k_\parallel^2)}{(k_F^2 - k_\parallel^2 + \beta^2)^2} \quad (52)$$

Evaluating this integral, we get $A_r \simeq (L^2\beta^2/\pi)F(\beta)$ where

$$F(\beta) \simeq \ln \frac{k_F^2 + \beta^2}{\beta^2} - \frac{k_F^2}{k_F^2 + \beta^2} \quad (53)$$

Inserting Eqs.(52) and (53) into Eq.(44) and expressing β with use of Eqs.(47) and (48), we obtain

$$\alpha' \simeq \frac{\gamma(mSJn_s)^2}{8\pi^2\hbar M_s d} F(\beta) \quad (54)$$

We can bring this result closer to the form of Eq.(30) by letting $n_s = a^{-2}$, and $M_s = \gamma S a^{-3}$ yielding

$$\gamma S^2 n_s^2 = \frac{M_s^2 a^2}{\gamma} \quad (55)$$

Furthermore, assuming $\beta \ll k_F$, the function $F(\beta)$ can be approximated by

$$F(\beta) \approx 2 \ln \frac{k_F}{1.65\beta} \approx 2 \ln \frac{\epsilon_F}{J_{sd}} \quad (56)$$

From Eqs.(54-56) we have

$$G' = \gamma M_s \alpha' \approx \frac{(JM_s a m)^2}{4\pi^2 \hbar d} \ln \frac{\epsilon_F}{J_{sd}} \quad (57)$$

This equation shows a remarkable similarity with the expression (30). Note, however, that the logarithmic terms do not match. If we consider an infinite system, and ignore the cutoff due to the magnetic length (32), the gap Δ vanishes, and Eq.(30) becomes logarithmically divergent. On the other hand, Eq.(57) shows that there is an effective gap, $\Delta \approx J_{sd}$ corresponding to a finite cutoff $q_1 \approx k_F(J_{sd}/\epsilon_F)$. Thus, the spin-pumping theory is infrared-divergence free. For real finite size systems, this difference is only of academic interest since the cutoff produced by the boundary conditions, $q_1 \approx D^{-1}$, yields a logarithmic term that is of the same order of magnitude as that in eq.(57).

The presence of an effective gap, $\Delta \approx J_{sd}$, in the spin pumping theory is presumably linked to the fact that, in contrast to linear response theory, it is of infinite order in the coupling constant J . This is seen in Eq.(54) where $F(\beta)$ is a nonlinear function of β given in Eq.(53). This kind of nonlinearity is generic in the scattering approach to transport (see Eq.(51)). In fact, Bruno [13] derives an exact expression for the static RKKY coupling that goes beyond the linear response result of Yafet [15].

V. DISCUSSION

Our numerical estimate of G' based on Eq.(38) suggests that a substantial enhancement of the FMR linewidth, that is independent of the atomic number Z , should be observed in $N/F/N$ systems. In contrast, the data of Mizukami et al.[8] on trilayers containing permalloy films show a strong dependence of G' on Z . In fact, for Cu , which has the smallest Z of the N -metals studied there is a complete absence of a $1/d$ -dependent G' . Tserkovnyak et al. [11] propose that it is the spin accumulation in the N -layer that is responsible for such a suppression of the ferromagnetic relaxation in copper layer. Note that the theory of spin-pumping assumes at the outset that the spin system in the N -layer is kept in thermal equilibrium during the precession. For that one needs an efficient spin-sink mechanism. The data of Ref.[8] indicate that spin-lattice relaxation via spin-orbit coupling [21] provides the required spin sink. In fact, metals with larger Z exhibit generally larger measured values of G' . This trend is in agreement with the fact that the spin-lattice relaxation rate scales as Z^4 [22].

Also the theory of G' , presented in Sec.III, assumes that the electron spins in the N -layers are in thermal equilibrium. This can be established either by the spin-lattice relaxation in the bulk, or by surface relaxation. One way to include these effects into the reaction field of Eq.(3) is to calculate the quantity $\langle \mathbf{s}(0, t) \rangle$ using the Bloch equation with diffusion [23]. This equation is to be solved with the b.c. that allows the electron spin to be flipped upon collision with the surface. Such b.c. has been proposed by Dyson [23]. In terms of the spin density $\langle \mathbf{s} \rangle$ this so called "evaporation" b.c. reads

$$\frac{\partial \langle \mathbf{s} \rangle}{\partial x} = \frac{3p}{4\Lambda} \langle \mathbf{s} \rangle \quad (58)$$

where p is the probability of spin flip to take place upon the reflection from the boundary, and Λ is the mean-free path in the bulk. Due to surface irregularities and paramagnetic surface impurities, the probability p can be large enough to provide the necessary spin sink even for layers with small bulk disorder.

Alternatively, the spin density and the reaction field can be obtained from the time-dependent 2×2 matrix kinetic equations driven by precessing magnetization of the ferromagnet [24]. Such an approach is inspired by the work of Kamberský [25] on intrinsic damping due to spin-orbit coupling in bulk ferromagnets. This work invokes the idea of a "breathing Fermi surface": The chemical potential varies in response to the time-dependent perturbation. However, the distribution of the electrons does not respond instantaneously to the perturbation. There is a time-lag characterized by a relaxation time τ . In Ref.[18] we apply this idea to the case of dynamic interlayer exchange coupling. In this case it is the spacer electrons which are affected by the spin-dependent potential at the interfaces, and the time variation of this potential is due to the precession of the ferromagnetic

moment. Thus, the relevant relaxation time is the transverse spin-relaxation time τ_{spin} . The resulting effective damping field, is like Eq.(20), proportional to $d\mathbf{M}(t)/dt$ implying Gilbert damping. However, in distinction from Eq.(20), it is also proportional to τ_{spin} .

This brings us to the question: Is there a system to which the ballistic theory of the present paper is applicable? We believe that the double-layer structure studied in Ref.[9] is a good example of such a system. Here, the precessing layer F_1 deposits spin current into the N -spacer and the second layer F_2 acts as an absorber of the transverse component of the spin current - thus providing an effective spin sink. Detailed analysis of this mechanism has been presented in beautiful papers by Stiles and Zangwill [26,27]. These authors show that there is an oscillatory, power law, decay of the transmitted transverse spin current that is caused by cancellations due to a distribution of precessional frequencies, and the rotation of the spin of the incoming spin upon reflection. Consequently, almost complete cancellation of the transverse spin takes place after propagation into the ferromagnet by a few lattice constants. This finding also supports our assumption that the excitation of transverse components of $\langle \mathbf{s}(x, t) \rangle$, via $s-d$ exchange, is confined to the N/F interface layer (see Appendix A).

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APPENDIX A: DERIVATION OF EQ.(3)

We consider a trilayer shown in Fig.1., and derive the reaction field from the torque equation

$$\left[\mathbf{M}_f(t) \times \mathbf{H}_r(t) \right] = \mathbf{T}(t) \quad (A1)$$

where $\mathbf{M}_f(t)$ is the net magnetic moment of the ferromagnetic film, and $\mathbf{T}(t)$ is the torque due to the $s-d$ interaction. Since only interface regions contribute to this torque (see Refs.[13] and [27]), we pick a magnetic atom in the plane $x=0$, and consider the local magnetic field $\mathbf{H}^{(i)}(t)$ acting on its magnetic moment $\mathbf{M}^{(i)}(t)$. Using Eq.(5), the expectation value of the $s-d$ exchange energy of this atom is $-\mathbf{J}\mathbf{S}^{(i)}(t) \cdot \langle \mathbf{s}(0, t) \rangle$. If we write this quantity as $-\mathbf{M}^{(i)}(t) \cdot \mathbf{H}^{(i)}(t)$, where $\mathbf{M}^{(i)}(t) = \gamma \mathbf{S}^{(i)}(t)$, the local magnetic field is

$$\mathbf{H}^{(i)}(t) = \frac{J}{\gamma} \langle \mathbf{s}(0, t) \rangle \quad (A2)$$

For a square film of area L^2 , the number of interface atoms is L^2/a^2 . Thus, using Eq.(A2), the net torque contributed by both interfaces is

$$\mathbf{T} = \frac{2L^2J}{a^2\gamma} \left[\mathbf{M}^{(i)}(t) \times \langle \mathbf{s}(0,t) \rangle \right] \quad (\text{A3})$$

Noting that the net magnetic moment of the film is $\mathbf{M}_f = \mathbf{M}^{(i)}L^2d/a^3$, and using Eq.(A3) in (A1), yields Eq.(3). Similar approach has been used to deduce the effective field in ultrathin layers in the presence of interfaces (see Eq.(1.6) in Ref.[3]).

APPENDIX B: SPIN RELAXATION AND INFRARED CUTOFF

To include spin relaxation into the theory of G' , we start from Eq.(22) and replace the infinitesimal quantity η by $\Gamma = \hbar/\tau_s$, where τ_s is the spin-relaxation time. Thus, the electron-hole pairs with flipped spin are assumed to relax with a frequency Γ/\hbar . Similar assumption has been made in the theory of magnon relaxation via $s-d$ interaction [28]. Moreover, we neglect the splitting Δ . Making these changes in Eqs.(22)-(24), we obtain

$$\text{Im}\chi_T(q, \omega) \approx -\frac{\hbar^2\Gamma}{16\pi^2} \int_0^\infty dk k^2 \frac{\partial f}{\partial \epsilon_k} I(k, q) \quad (\text{B1})$$

where

$$I(k, q) = \int_0^\pi d\theta \sin \theta \frac{\hbar^2 k q \cos \theta / m}{(\hbar\omega - \hbar^2 k q \cos \theta / m)^2 + \Gamma^2} \quad (\text{B2})$$

Expanding the integrand to order ω , the θ -integration yields

$$I(k, q) = \frac{2\hbar\omega}{\Gamma^2 x} \left[\tan^{-1} x - \frac{x}{1+x^2} \right] \quad (\text{B3})$$

where $x = \hbar^2 k q / (\Gamma m)$. Using Eq.(B3), the k -integration in Eq.(B1) yields at $T = 0$

$$\text{Im}\chi_T(q, \omega) \approx \frac{m^2\omega}{8\pi^2\hbar q} \left[\tan^{-1}(v_F\tau_s q) - \frac{v_F\tau_s q}{1 + (v_F\tau_s q)^2} \right] \quad (\text{B4})$$

Consistent with Eq.(27), this expression is equal to $m^2\omega/(16\pi\hbar q)$ in the limit $\tau_s \rightarrow \infty$. The evaluation of the q -integral of this expression is done by approximating $\tan^{-1} x$ by $\pi x/2$ for $x < 1$, and by $\pi/2$ for $x > 1$. For $\epsilon_F \gg \Gamma$, we obtain

$$\int_{-q_2}^{q_2} \frac{dq}{2\pi} \text{Im}\chi_T(q, \omega) \approx \frac{m^2\omega}{16\pi^2\hbar} \ln(v_F\tau_s q_2) \quad (\text{B5})$$

Introducing this result into Eq.(21), the damping enhancement in the presence of spin relaxation is given by

$$G' \approx \frac{(JM_s a m)^2}{8\pi^2\hbar d} \ln \frac{q_2}{q_1} \quad (\text{B6})$$

where $q_1 \approx (v_F\tau_s)^{-1}$ is the infrared cutoff mentioned at the end of Sec.IIIA.

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FIG. 1: A trilayer consisting of normal metals(N) adjacent to a ferromagnetic film (F) of thickness d . The $s-d$ interaction generating the spin density \mathbf{s} is assumed to take place in contact layers of thickness, a , of the order of lattice constant.

